An Efficient Mixed Algorithm of L-MEI and DDM for the Wave Scattering by a Concave Cylinder

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***Abstract*—In this paper, a localized MEI method (L-MEI) is developed and combined with the domain decomposition method (DDM) [9] for the simulation of scattering by a concave cylinder. In the L-MEI, the whole domain is decomposed into many subdomains. Different from the conventional MEI method, the MEI coefficients of the L-MEI method in each subdomain are only dependent on the localized metrons that are defined in the subdomain. The localization of metrons has the following advantages: 1) speeding up the calculation of MEI coefficients and saving memory, 2) making the MEI method available for concave structures, and 3) obtaining a band sparse matrix directly without any modification.**

***Index Terms*—Domain decomposition method, electromagnetic scattering, measured equation of invariance.**

I. Introduction

As a very efficient technique, the measured equation of the invariance (MEI) method has been applied to many electromagnetic (EM) problems [1]–[7]. Recently, some fast algorithms were adopted to solve the final sparse matrix equation [3], [4], and then the calculating time for solving the matrix equation was reduced from $O(N^2)$ to $O(N)$, where N is the order of the sparse matrix. However, the computation complexity for the calculation of MEI coefficients remains $O(N^2)$, and it has become a bottle-neck for scattering problems. Although many efforts have been made to accelerate the calculation of MEI coefficients that save the calculating time by one or two orders of magnitude [5]–[7], the more efficient calculation of MEI coefficients is still necessary. On the other hand, when the objects are concave, the concave region must be completely fill with meshes in the MEI method for avoiding poor results, which decreases the efficiency of the MEI method because the final sparse matrix is not a banded-like matrix. The above problems result from the fact that the MEI at each truncated boundary node is dependent on the metrons which are defined along the whole boundary of the object.

In this paper, a localized MEI method (L-MEI) is developed and combined with the domain decomposition method (DDM) [8] for the simulation of EM scattering by a concave cylinder. In the L-MEI, the whole boundary domain of the object is decomposed into many subdomains. Different from the conventional MEI method, the MEI coefficients of the L-MEI method in each subdomain are only dependent on the localized metrons that are defined in the subdomain. The localization of metrons has some advantages: 1) speeding up the calculation of MEI coefficients and saving memory, 2) making the MEI method available for concave structures, and 3) obtaining a band sparse matrix directly without any modifications such as in [3], [6].

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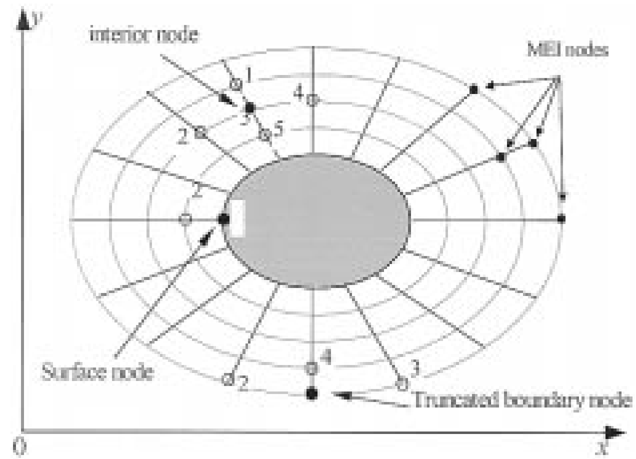

Fig. 1. Conducting cylinder geometry and conformal mesh.

II. Formulation

The cross-section of an infinitely long conducting cylinder and the conformal meshes around the cylinder are illustrated in Fig. 1. TM^z incident wave is assumed. Denoting the longitude component of the electric field by ϕ, the total field on each node consists of incident and scattered ones with the relation as $\phi = \phi^{inc} + \phi^s$. The finite difference equations for interior nodes may be expressed as

$$\sum_{i=1}^{5} a_i \phi_i^s = 0 \tag{1}$$

where the expressions for the coefficients a_i can be found in [2].

With the surface impedance boundary condition (SIBC), we have finite difference equations for nodes on the surface of the cylinder as

$$\phi_1 + b_2\phi_2^s = -b_2\phi_2^{inc} \tag{2}$$

where b_i can be found in [4].

For the nodes on the truncated boundary, the MEI is available

$$\sum_{j=1}^{4} c_j \phi_j^s = 0 \tag{3}$$

where c_j are the MEI coefficients and will be determined by metrons that are defined on the whole surface of the cylinder. To calculate the MEI coefficients at a truncated boundary node,

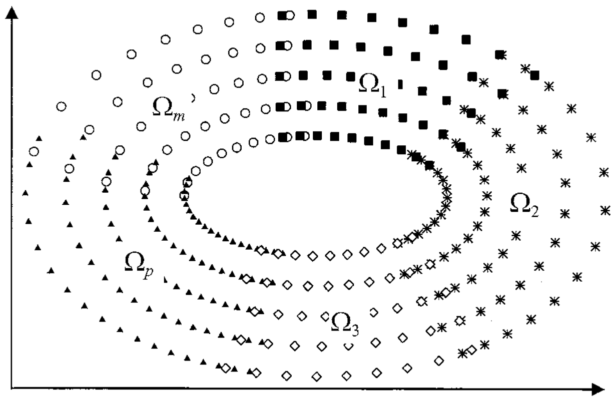

Fig. 2. Domain decomposition along the circumference.

we need to know the scattered fields at the MEI nodes produced by the metrons which can be calculated by the following integral operation

$$\phi^s\left(\overrightarrow{r}_j\right) = -\int_\Gamma 30\pi k_0 \psi(l') H_0^{(2)}\left(k_0\left|\overrightarrow{r}_j - \overrightarrow{r}'_i\right|\right) dl'. \quad (4)$$

The definitions of the parameters in (4) can be found in [2]. Usually, the metrons may be chosen as

$$\psi(l') = \begin{cases} \cos(2\pi n l'/L) \\ \sin(2\pi n l'/L) \end{cases} \qquad n = 0, 1, 2, \ldots \quad (5)$$

where L is the perimeter of the cylinder, and l' is the arc length along the surface of the cylinder. Because the metrons are defined on the whole surface of the cylinder, the calculation of the integral in (4) by a direct summation method is very time-consuming. To localize the metrons, we decompose the original region Ω into several subdomains $\Omega_p (p = 1, 2, \ldots m)$, which is shown in Fig. 2. Let $\Gamma_{p,q} = \partial\Omega_p \cap \partial\Omega_q$ be the common boundary between sub-domain Ω_p and Ω_q. Using the domain decomposition method [8], we have an iterative algorithm for ϕ_p in the splitted domain

$$\begin{aligned} &\forall n \geq 0 \quad \text{and} \quad p = 1, 2, \ldots m \\ &\nabla^2 \phi_p^{n+1} + k_0^2 \phi_p^{n+1} = 0 && \text{in } \Omega_p \\ &\left(\partial_{\nu_p} + jk_0\right)\phi_p^{n+1} \\ &\quad = \left(-\partial_{\nu_q} + jk_0\right)\phi_q^n && \text{on } \Gamma_{pq},\ \forall q \neq p \\ &\text{MEI or SIBC} && \text{on } \Gamma_p \end{aligned} \quad (6)$$

where $\Gamma_p = \partial\Omega \cap \partial\Omega_p$ is the common boundary between Ω and Ω_p and ϕ_p^n is the approximate solution of ϕ_p in Ω_p at the nth iteration.

Because the metrons in a subdomain are only defined on the surface of the subdomain, the MEI coefficients in the subdomain are independent of the metrons in all other subdomains. The localization of metrons decreases the calculating time for MEI coefficients to about $1/m$, where m is the number of subdomains. On the other hand, because each subdomain is not closed, so the final coefficient matrices derived from these subdomains are band sparse matrices. The computation complexity for a band matrix equation is $O(N_p)$, where N_p is the number of unknowns in the pth subdomain. In addition, each subdomain can be calculated independently, so the maximum memory cost can be decreased to $O(N_{\max})$, and $N_{\max} = \max(N_p)$.

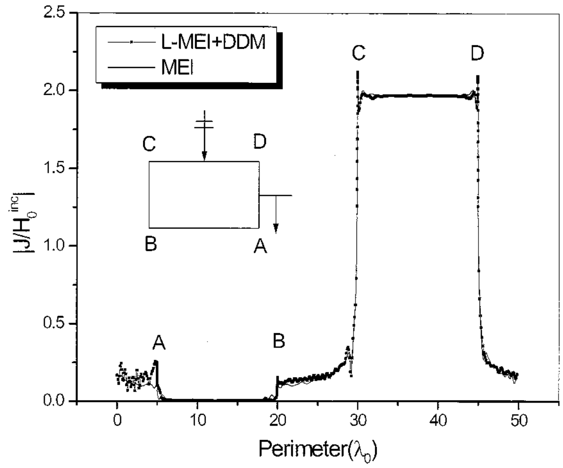

Fig. 3. Comparison of the surface current of a rectangular cylinder.

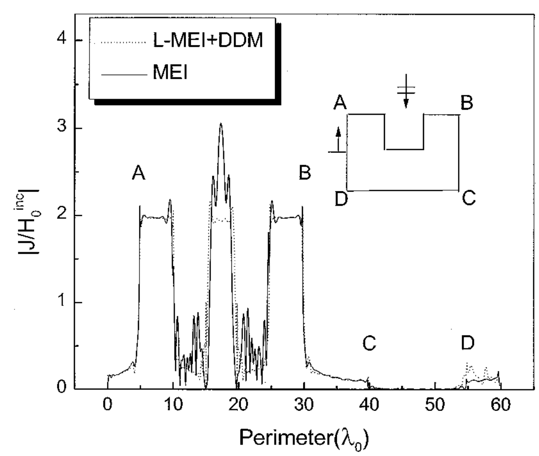

Fig. 4. Comparison of the surface currents of a cylinder with a groove.

III. Numerical Results

Using both the L-MEI + DDM method and the conventional MEI method, we have analyzed EM scattering problems of a rectangular conducting cylinder and a conducting cylinder with a groove, and the calculated surface currents are shown in Figs. 3 and 4, respectively. In the conventional MEI method, the region inside the groove of the concave cylinder must be filled with meshes for a good result. The L-MEI + DDM is beyond the limitation, and the conformal meshes are filled only along the boundary of the cylinder. Here, the number of subdomains in the DDM is chosen as 6, and the currents have converged after 3 iterations. It can be seen that the surface currents simulated with the L-MEI + DDM method and the conventional MEI method are in reasonable agreement.

IV. Conclusions

By combining the L-MEI method and the DDM, this paper presents an efficient approach for the simulation of EM scattering by a concave cylinder. The L-MEI method not only saves the computing time, but also provides a base to be combined with DDM, which makes the MEI method available to more complex structures, such as a concave cylinder. Although the DDM was used in the paper, the L-MEI can be used alone for the simulation of EM scattering by convex objects without use of the DDM.

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